Mark Scheme Notes

- Marks are of the following three types:

  M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

  A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

  B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

- The following abbreviations may be used in a mark scheme or used on the scripts:

  AEF Any Equivalent Form (of answer is equally acceptable)

  AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

  BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

  CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

  CWO Correct Working Only – often written by a 'fortuitous' answer

  ISW Ignore Subsequent Working

  MR Misread

  PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

  SOS See Other Solution (the candidate makes a better attempt at the same question)

  SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √"marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

- PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

June 2003

GCE A AND AS LEVEL

MARK SCHEME

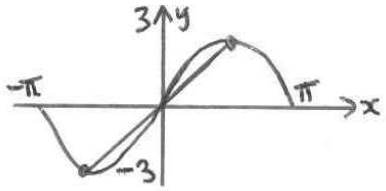
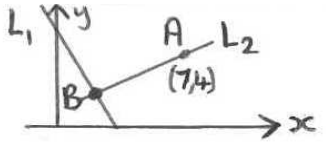
MAXIMUM MARK: 75

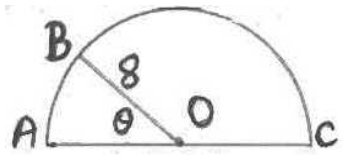
SYLLABUS/COMPONENT: 9709/01

MATHEMATICS
Paper 1 (Pure 1)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1. $(2x - 1/x)^5$. 4^{th} term needed. $\rightarrow {}_5C_3 = 5.4/2$ $\rightarrow x\ 2^2\ x\ (-1)^3$ $\rightarrow -40$	M1 DM1 A1 [3]	Must be 4^{th} term – needs $(2x)^2\ (1/x)^3$ Includes and converts ${}_5C_2$ or ${}_5C_3$ Co Whole series given and correct term not quoted, allow 2/3
2. $\sin 3x + 2\cos 3x = 0$ $\tan 3x = -2$ $x = 38.9$ (8) and $x = 98.9$ (8) and $x = 158.9$ (8) NB. $\sin^2 3x + \cos^2 3x = 0$ etc. M0 But $\sin^2 3x = (-2\cos 3x)^2$ plus use of $s^2 + c^2 = 1$ is OK Alt. $\sqrt{5}\sin(3x + \alpha)$ or $\sqrt{5}\cos(3x - \alpha)$ both OK	M1 A1 A1√ A1√ [4]	Use of tan = sin ÷ cos with 3x Co For 60 + "his" For 120 + "his" and no others in range (ignore excess ans. outside range) Loses last A mark if excess answers in the range
3. (a) $dy/dx = 4 - 12x^{-3}$ (b) $\int = 2x^2 - 6x^{-1} + c$ (a) (quotient OK M1 correct formula, A1 co)	B2, 1 [2] 3 x B1 [3]	One off for each error (4, -, 12, -3) One for each term – only give +c if obvious attempt at integration
4. $a = -10$ $a + 14d = 11$ $d = \frac{3}{2}$ $a + (n - 1)d = 41$ $n = 35$ Either $S_n = n/2(2a + (n - 1)d)$ or $n/2(a + l)$ $= 542.5$	M1 M1 A1 M1 A1 [5]	Using $a = (n - 1)d$ Correct method – not for $a + nd$ Co Either of these used correctly For his d and any n
5. (i) $2a + b = 1$ and $5a + b = 7$ $\rightarrow a = 2$ and $b = -3$ (ii) $f(x) = 2x - 3$ $ff(x) = 2(2x - 3) - 3$ $\rightarrow 4x - 9$ $= 0$ when $x = 2.25$	M1 A1 [2] M1 DM1 A1 [3]	Realising how one of these is formed Co Replacing "x" by "his ax + b" and "+b" For his a and b and solved = 0 Co

6.	(i) 	B2, 1 [2]	For complete cycle, shape including curves, not lines, -3 to +3 shown or implied, for -π to π. Degrees ok
	(ii) x = π/2, y = 3 (allow if 90°) → k = 6/π co.	M1 A1 [2]	Realising maximum is (π/2, 3) + sub Co (even if no graph)
	(iii) (-π/2, -3) – must be radians	B1 [1]	Co (could come from incorrect graph)
7.	(i) 		
	Gradient of L_1 = -2 Gradient of L_2 = ½ Eqn of L_2 y – 4 = ½(x – 7)	B1 M1 M1A1√ [4]	Co – anywhere Use of m_1m_2 = -1 Use of line eqn – or y = mx + c. Line must be through (7, 4) and non-parallel
	(ii) Sim Eqns → x = 3, y = 2	M1 A1	Solution of 2 linear eqns Co
	AB = $\sqrt{(2^2 + 4^2)}$ = √20 or 4.47	M1A1 [4]	Correct use of distance formula. Co
8.	(i) $\overrightarrow{BA}$ = **a – b** = **i** + 2**j** – 3**k** $\overrightarrow{BC}$ = **c – b** = -2**i** + 4**j** + 2**k** Dot product = -2 + 8 – 6 = 0 → Perpendicular	M1 M1A1 A1 [4]	Knowing how to use position vector for $\overrightarrow{BA}$ or $\overrightarrow{BC}$ – not for $\overrightarrow{AB}$ or $\overrightarrow{CB}$ Knowing how to use $x_1y_1 + x_2y_2 + x_3y_3$. Co Correct deduction. Beware fortuitous (uses $\overrightarrow{AB}$ or $\overrightarrow{CB}$ – can get 3 out of 4)
	(ii) $\overrightarrow{BC}$ = **c – b** = -2**i** + 4**j** + 2**k** $\overrightarrow{AD}$ = **d – a** = -5**i** + 10**j** + 5**k** These are in the same ratio \ parallel	M1 M1	Knowing how to get one of these Both correct + conclusion. Could be dot product = 60 → angle = 0°
	Ratio = 2:5 (or √24: √150)	M1A1 [4]	Knowing what to do. Co. Allow 5:2

9.			
	(i) θ = 1 angle BOC = π-θ Area = ½r²θ = 68.5 or 32(π-1) (or ½circle-sector)	B1 M1 A1 [3]	For π-θ or for ½πr² – sector Use of ½r²θ Co NB. 32 gets M1 only
	(ii) 8 + 8 + 8θ = ½(8 + 8 + 8(π-θ)) Solution of this eqn → 0.381 or $^1/_3$(π-2)	M1 M1 A1 [3]	Relevant use of s = rθ twice Needs θ – collected – needs perimeters Co.
	(iii) θ = π/3 AB = 8cm BC = 2 x 8sinπ/3 = 8√3 Perimeter = 24 + 8√3	B1 M1 A1 [3]	Co. Valid method for BC – cos rule, Pyth allow decimals here Everything OK. Answer given NB. Decimal check loses this mark
10.	y = √(5x + 4)		
	(i) dy/dx = ½(5x + 4)$^{-½}$ x 5 x = 1, dy/dx = 5/6	B1B1 B1 [3]	½(5x + 4)$^{-½}$ x 5 B1 for each part Co
	(ii) dy/dt = dy/dx x dx/dt = 5/6 x 0.03 → 0.025	M1 A1√ [2]	Chain rule correctly used For (i) x 0.03
	(iii) realises that area → integration $\int$ = (5x + 4)$^{3/2}$ ÷ $^3/_2$ ÷ 5 Use of limits → 54/15 - 16/15 = 38/15 = 2.53	M1 A1A1 DM1 A1 [5]	Realisation + attempt – must be (5x + 4)^k For (5x + 4)$^{3/2}$ ÷ $^3/_2$. For ÷ 5 Must use "0" to "1" Co

11.	(i) $8x - x^2 = a - x^2 - b^2 - 2bx$ + equating $\rightarrow$ b = -4 $a = b^2 = 16$ (i.e. $16 - (x - 4)^2$)	M1 B1 A1 [3]	Knows what to do – some equating Anywhere – may be independent For 16- ($)^2$
	(ii) $dy/dx = 8 - 2x = o$ when $\rightarrow$ (4, 16) (or from –b and a)	M1 A1 [2]	Any valid complete method Needs both values
	(iii) $8x - x^2 \geq -20$ $x^2 - 8x - 20 = (x - 10)(x + 2)$ End values –2 and 10 Interval $-2 \leq x \leq 10$	M1 A1 A1 [3]	Sets to 0 + correct method of solution Co – independent of < or > or = Co – including $\leq$ (< gets A0)
	g: $x \rightarrow 8x - x^2$ for $x \geq 4$		
	(iv) domain of g^{-1} is $x \leq 16$ range of g^{-1} is $g^{-1} \geq 4$	B1√ B1 [2]	From answer to (i) or (ii). Accept <16 Not f.t since domain of g given
	(v) $y = 8x - x^2 \rightarrow x^2 - 8x + y = 0$	M1	Use of quadratic or completed square expression to make x subject
	$x = 8 \pm \sqrt{(64 - 4y)} \div 2$ $g^{-1}(x) = 4 + \sqrt{(16 - x)}$	DM1 A1 [3]	Replaces y by x Co (inc. omission of -)
	or $(x - 4)^2 = 16 - y \rightarrow x = 4 + \sqrt{(16 - y)}$ $\rightarrow y = 4 + \sqrt{(16 - x)}$		

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

June 2003

GCE AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/02

MATHEMATICS
Paper 2 (Pure 2)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1 *EITHER:* State or imply non-modular inequality $(x - 4)^2 > (x + 1)^2$, or corresponding equation — B1
Expand and solve a linear inequality, or equivalent — M1
Obtain critical value 1½ — A1
State correct answer $x < 1½$ (allow $\leq$) — A1

OR: State a correct linear equation for the critical value e.g. $4 - x = x + 1$ — B1
Solve the linear equation for x — M1
Obtain critical value 1½, or equivalent — A1
State correct answer $x < 1½$ — A1

OR: State the critical value 1½, or equivalent, from a graphical method or by inspection or by solving a linear inequality — B3
State correct answer $x < 1½$ — B1

[4]

2 (i) *EITHER:* Expand *RHS* and obtain at least one equation for a — M1
Obtain $a^2 = 9$ and $2a = 6$, or equivalent — A1
State answer $a = 3$ only — A1

OR: Attempt division by $x^2 + ax + 1$ or $x^2 - ax - 1$, and obtain an equation in a — M1
Obtain $a^2 = 9$ and either $a^3 - \text{l l}a + 6 = 0$ or $a^3 - 7a - 6 = 0$, or equivalent — A1
State answer $a = 3$ only — A1

[Special case: the answer $a = 3$, obtained by trial and error, or by inspection, or with no working earns B2.] **[3]**

(ii) Substitute for a and attempt to find zeroes of one of the quadratic factors — M1
Obtain one correct answer — A1
State all four solutions ½(-3 ± $\sqrt{5}$) and ½(3 ± $\sqrt{13}$), or equivalent — A1

[3]

3 (i) State or imply indefinite integral of e^{2x} is ½e^{2x}, or equivalent — B1
Substitute correct limits correctly — M1
Obtain answer R = ½ e^{2p} - ½, or equivalent — A1

[3]

(ii) Substitute $R = 5$ and use logarithmic method to obtain an equation in $2p$ — M1*
Solve for p — M1 (dep*)
Obtain answer $p = 1.2$ (1.1989 ...) — A1

[3]

4 (i) Use tan ($A \pm B$) formula to obtain an equation in tan x M1

State equation $\frac{\tan x + 1}{1 - \tan x} = 4\frac{(1 - \tan x)}{1 + \tan x}$, or equivalent A1

Transform to a 2- or 3-term quadratic equation M1

Obtain given answer correctly A1

[4]

(ii) Solve the quadratic and calculate one angle, or establish that $t = {}^1/_3$, 3 (only) M1

Obtain one answer, e.g. $x = 18.4^\circ \pm 0.1^\circ$ A1

Obtain second answer $x = 71.6^\circ$ and no others in the range A1

[Ignore answers outside the given range] **[3]**

5 (i) Make recognizable sketch over the given range of two suitable graphs, e.g. $y = \ln x$ and $y = 2 - x^2$ B1+B1

State or imply link between intersections and roots and justify given answer B1

[3]

(ii) Consider sign of $\ln x - (2 - x^2)$ at $x = 1$ and $x = 1.4$, or equivalent M1

Complete the argument correctly with appropriate calculation A1

[2]

(iii) Use the given iterative formula correctly with $1 \leq x_n \leq 1.4$ M1

Obtain final answer 1.31 A1

Show sufficient iterations to justify its accuracy to 2d.p., or show there is a sign change in the interval (1.305, 1.315) A1

[3]

6 (i) Attempt to apply the chain or quotient rule M1

Obtain derivative of the form $\frac{k\sec^2 x}{(1 + \tan x)^2}$ or equivalent A1

Obtain correct derivative $-\frac{\sec^2 x}{(1 + \tan x)^2}$ or equivalent A1

Explain why derivative, and hence gradient of the curve, is always negative A1

[4]

(ii) State or imply correct ordinates: 1, 0.7071.., 0.5 B1

Use correct formula, or equivalent, with $h = {}^1/_8\pi$ and three ordinates M1

Obtain answer 0.57 (0.57220...) $\pm$ 0.01 (accept 0.18π) A1

[3]

(iii) Justify the statement that the rule gives an over-estimate B1

[1]

7 (i) State $\frac{dx}{d\theta} = 2 - 2\cos 2\theta$ or $\frac{dy}{d\theta} = 2\sin 2\theta$ B1

Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$ M1

Obtain answer $\frac{dy}{dx} = \frac{2\sin 2\theta}{2 - 2\cos 2\theta}$ or equivalent A1

Make relevant use of sin 2*A* and cos 2*A* formulae (indep.) M1

Obtain given answer correctly A1

[5]

(ii) Substitute $\theta = ¼\pi$ in $\frac{dy}{dx}$ and both parametric equations M1

Obtain $\frac{dy}{dx} = 1$, $x = ½\pi - 1$, $y = 2$ A1

Obtain equation $y = x + 1.43$, or any exact equivalent A1√

[3]

(iii) State or imply that tangent is horizontal when $\theta = ½\pi$ or $^3/_2\pi$ B1

Obtain a correct pair of *x* , *y* or *x*- or *y*-coordinates B1

State correct answers (π, 3) and (3π, 3) B1

[3]

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

June 2003

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 75

SYLLABUS/COMPONENT: 9709/03, 8719/03

MATHEMATICS AND HIGHER MATHEMATICS
Paper 3 (Pure 3)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1 (i) Use trig formulae to express *LHS* in terms of sin *x* and cos *x* M1
Use cos 60° = sin 30° to reduce equation to given form cos *x* = *k* M1

[2]

(ii) State or imply that $k = -\frac{1}{\sqrt{3}}$ (accept -0.577 or -0.58) A1
Obtain answer x = 125.3° only A1
[Answer must be in degrees; ignore answers outside the given range.]
[SR: if $k = \frac{1}{\sqrt{3}}$ is followed by *x* = 54.7°, give A0A1√.]

[2]

2 State first step of the form $kxe^{2x} \pm \int ke^{2x}\,dx$ M1
Complete the first step correctly A1
Substitute limits correctly having attempted the further integration of ke^{2x} M1
Obtain answer ¼ (e^2 + 1) or exact equivalent of the form $ae^2 + b$, having used e^0 =1 throughout A1

[4]

3 *EITHER* State or imply non-modular inequality $(x-2)^2 < (3-2x)^2$, or corresponding equation B1
Expand and make a reasonable solution attempt at a 2- or 3-term quadratic, or equivalent M1
Obtain critical value *x* = 1 A1
State answer *x* < 1 only A1

OR State the relevant linear equation for a critical value, i.e. 2 - *x* = 3 - 2*x*, or equivalent B1
Obtain critical value *x* = 1 B1
State answer *x* < 1 B1
State or imply by omission that no other answer exists B1

OR Obtain the critical value *x* = 1 from a graphical method, or by inspection, or by solving a linear inequality B2
State answer *x* < 1 B1
State or imply by omission that no other answer exists B1

[4]

4 (i) *EITHER* State or imply that x - 2 is a factor of f(x) B1
Substitute 2 for x and equate to zero M1
Obtain answer a = 8 A1

[The statement $(x-2)^2 = x^2 - 4x + 4$ earns B1.]

OR Commence division by $x^2 - 4x + 4$ and obtain partial quotient $x^2 + 2x$ B1
Complete the division and equate the remainder to zero M1
Obtain answer a = 8 A1

OR Commence inspection and obtain unknown factor $x^2 + 2x + c$ B1
Obtain $4c = a$ and an equation in c M1
Obtain answer a = 8 A1

[3]

(ii) *EITHER* Substitute a = 8 and find other factor $x^2 + 2x + 2$ by inspection or division B1
State that $x^2 - 4x + 4 \geq 0$ for all x (condone > for $\geq$) B1
Attempt to establish sign of the other factor M1
Show that $x^2 + 2x + 2 > 0$ for all x and complete the proof A1
[An attempt to find the zeros of the other factor earns M1.]

OR Equate derivative to zero and attempt to solve for x M1
Obtain $x = -\frac{1}{2}$ and 2 A1
Show correctly that f(x) has a minimum at each of these values A1
Having also obtained and considered x = 0, complete the proof A1

[4]

5 (i) State or imply $w = \cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi$ (allow decimals) B1

Obtain answer $uw = -\sqrt{3} - i$ (allow decimals) B1√

Multiply numerator and denominator of $\frac{u}{w}$ by $-1 - i\sqrt{3}$, or equivalent M1

Obtain answer $\frac{u}{w} = \sqrt{3} - i$ (allow decimals) A1

[4]

(ii) Show U on an Argand diagram correctly B1
Show A and B in relatively correct positions B1√

[2]

(iii) Prove that $AB = UA$ (or UB), or prove that angle AUB = angle ABU (or angle BAU) or prove, for example, that $AO = OB$ and angle $AOB = 120°$, or prove that one angle of triangle UAB equals 60° B1
Complete a proof that triangle UAB is equilateral B1

[2]

6 (i) *EITHER* State or imply $f(x) \equiv \frac{A}{2x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$ B1

State or obtain $A = 1$ B1

State or obtain $C = 8$ B1

Use any relevant method to find B M1

Obtain value $B = 4$ A1

OR State or imply $f(x) \equiv \frac{A}{2x+1} + \frac{Dx+E}{(x-2)^2}$ B1

State or obtain $A = 1$ B1

Use any relevant method to find D or E M1

Obtain value $D = 4$ A1

Obtain value $E = 0$ A1

[5]

(ii) *EITHER* Use correct method to obtain the first two terms of the expansion of $(1 + 2x)^{-1}$ or $(x - 2)^{-1}$ or $(x - 2)^{-2}$ or $(1 - ½x)^{-1}$ or $(1 - ½x)^{-2}$ M1

Obtain any correct sum of unsimplified expansions up to the terms in x^2 (deduct A1 for each incorrect expansion) A2√

Obtain the given answer correctly A1

[Unexpanded binomial coefficients involving -1 or -2, e.g. $\binom{-2}{1}$ are not sufficient for the M1.]

[f.t. is on *A*, *B, C, D, E.*]

[Apply this scheme to attempts to expand $(9x^2 + 4)(1+2x)^{-1}(x - 2)^{-2}$, giving M1A2 for a correct product of expansions and A1 for multiplying out and reaching the given answer correctly.]

[Allow attempts to multiply out $(1 + 2x)(x - 2)^2 (1 - x + 5x^2)$, giving B1 for reduction to a product of two expressions correct up to their terms in x^2, M1 for attempting to multiply out as far as terms in x^2, A1 for a correct expansion, and A1 for obtaining $9x^2 + 4$ correctly.]

[SR: *B* or *C* omitted from the form of partial fractions. In part **(i)** give the first B1, and M1 for the use of a relevant method to obtain *A*, *B,* or *C*, but no further marks. In part **(ii)** only the M1 and A1√ for an unsimplified sum are available.]

[SR: *E* omitted from the form of partial fractions. In part **(i)** give the first B1, and M1 for the use of a relevant method to obtain *A* or *D*, but no further marks. In part **(ii)** award M1A2√A1 as in the scheme.]

OR Differentiate and evaluate f(0) and f′(0) M1

Obtain f(0) = 1 and f′(0) = -1 A1

Differentiate and obtain f″(0) = 10 A1

Form the Maclaurin expansion and obtain the given answer correctly A1

[4]

7 (i) State or imply that $\frac{dx}{dt} = k(100 - x)$ B1

Justify $k = 0.02$ B1

[2]

(ii) Separate variables and attempt to integrate $\frac{1}{100 - x}$ M1

Obtain term $-\ln(100 - x)$, or equivalent A1

Obtain term $0.02t$, or equivalent A1

Use $x = 5$, $t = 0$ to evaluate a constant, or as limits M1

Obtain correct answer in any form, e.g. $-\ln(100 - x) = 0.02t - \ln 95$ A1

Rearrange to give x in terms of t in any correct form, e.g. $x = 100 - 95\exp(-0.02t)$ A1

[6]

[SR: $\ln(100 - x)$ for $-\ln(100 - x)$. If no other error and $x = 100 - 95\exp(0.02t)$ or equivalent obtained, give M1A0A1M1A0A1√]

(iii) State that x tends to 100 as t becomes very large B1

[1]

8 (i) State derivative $\frac{1}{x} - \frac{2}{x^2}$, or equivalent B1

Equate 2-term derivative to zero and attempt to solve for x M1

Obtain coordinates of stationary point $(2, \ln 2 + 1)$, or equivalent A1+A1

Determine by any method that it is a minimum point, with no incorrect work seen A1

[5]

(ii) State or imply the equation $\alpha = \frac{2}{3 - \ln\alpha}$ B1

Rearrange this as $3 = \ln\alpha + \frac{2}{\alpha}$ (or *vice versa)* B1

[2]

(iii) Use the iterative formula correctly at least once M1

Obtain final answer 0.56 A1

Show sufficient iterations to justify its accuracy to 2 d.p., or show there is a sign change in the interval (0.555, 0.565) A1

[3]

9 (i) State or imply a correct normal vector to either plane, e.g. $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ or $2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$ B1

Carry out correct process for evaluating the scalar product of both the normal vectors M1

Using the correct process for the moduli, divide the scalar product of the two normals by the product of their moduli and evaluate the inverse cosine of the result M1

Obtain answer 40.4° (or 40.3°) or 0.705 (or 0.704) radians A1

[Allow the obtuse answer 139.6° or 2.44 radians]

[4]

(ii) *EITHER* Carry out a complete strategy for finding a point on l M1
Obtain such a point e.g. (0, 3, 2) A1

EITHER Set up two equations for a direction vector
$a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ of l, e.g. $a + 2b - 2c = 0$
and $2a - 3b + 6c = 0$ B1
Solve for one ratio, e.g. $a:b$ M1
Obtain $a:b:c = 6: -10: -7$, or equivalent A1
State a correct answer, e.g. $\mathbf{r} = 3\mathbf{j} + 2\mathbf{k} + \lambda(6\mathbf{i} - 10\mathbf{j} - 7\mathbf{k})$ A1√
OR Obtain a second point on l, e.g. (6, -7, -5) A1
Subtract position vectors to obtain a direction vector for l M1
Obtain $6\mathbf{i} - 10\mathbf{j} - 7\mathbf{k}$, or equivalent A1
State a correct answer, e.g. $\mathbf{r} = 3\mathbf{j} + 2\mathbf{k} + \lambda(6\mathbf{i} - 10\mathbf{j} - 7\mathbf{k})$ A1√
OR Attempt to find the vector product of the two normal vectors M1
Obtain two correct components A1
Obtain $6\mathbf{i} - 10\mathbf{j} - 7\mathbf{k}$, or equivalent A1
State a correct answer, e.g. $\mathbf{r} = 3\mathbf{j} + 2\mathbf{k} + \lambda(6\mathbf{i} - 10\mathbf{j} - 7\mathbf{k})$ A1√

OR Express one variable in terms of a second M1
Obtain a correct simplified expression, e.g. $x = (9 - 3y)/5$ A1
Express the same variable in terms of the third and form a three term equation M1
Incorporate a correct simplified expression, e.g. $x = (12 - 6z)/7$ in this equation A1
Form a vector equation for the line M1
State a correct answer, e.g. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ -5/3 \\ -7/6 \end{pmatrix} \lambda$, or equivalent A1√

OR Express one variable in terms of a second M1
Obtain a correct simplified expression, e.g. $y = (9 - 5x)/3$ A1
Express the third variable in terms of the second M1
Obtain a correct simplified expression, e.g. $z = (12 - 7x)/6$ A1
Form a vector equation for the line M1
State a correct answer, e.g. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -5/3 \\ -7/6 \end{pmatrix}$, or equivalent A1√

[6]

10 (i) *EITHER* Make relevant use of the correct sin 2*A* formula M1
Make relevant use of the correct cos 2*A* formula M1
Derive the given result correctly A1

OR Make relevant use of the tan 2*A* formula M1
Make relevant use of $1 + \tan^2 A = \sec^2 A$ or $\cos^2 A + \sin^2 A = 1$ M1
Derive the given result correctly A1

[3]

(ii) State or imply indefinite integral is ln sin x, or equivalent B1
Substitute correct limits correctly M1
Obtain given exact answer correctly A1

[3]

(iii) *EITHER* State indefinite integral of cos $2x$ is of the form k ln sin $2x$ M1
State correct integral ½ ln sin $2x$ A1
Substitute limits correctly throughout M1
Obtain answer ¼ 1n 3, or equivalent A1

OR State or obtain indefinite integral of cosec $2x$ is of the form k ln tan x, or equivalent M1
State correct integral ½ ln tan x, or equivalent A1
Substitute limits correctly M1
Obtain answer ¼ ln 3, or equivalent A1

[4]

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INTERNATIONAL EXAMINATIONS

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GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/04

MATHEMATICS
Paper 4 (Mechanics 1)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

Mechanics 1

1	**(i)**	Tension is 8000 N or 800*g* Accept 7840 N (from 9.8) or 7850 (from 9.81)	B1	1
	(ii)	For using $P = \frac{\Delta W}{\Delta t}$ or $P = Tv$	M1	
		$\Delta W = 8000 \times 20$ or $v = \frac{20}{50}$	A1 ft	
		Power applied is 3200 W Accept 3140 W (from 9.8 or 9.81)	A1	3
		SR (for candidates who omit *g*) (Max 2 out of 3) P = $800 \times 20 \div 50$ B1 Power applied is 320 W B1		
2	**(i) (a)**	For resolving in the direction *PQ*	M1	
		Component is 2 x 10cos30° – 6cos60° or 14.3 N or $10\sqrt{3} - 3$ N	A1	2
	(b)	Component is $\pm$6cos30° – 6cos60° or $\pm$5.20 N or $\pm 3\sqrt{3}$ N	B1	1
		SR (for candidates who resolve parallel to and perpendicular to the force of magnitude 6 N) (Max 2 out of 3) For resolving in both directions M1 For *X* = 6 – 10cos 30° or –2.66 N and *Y* = 10 + 10sin 30° or 15 N A1 **SR** (for candidates who give a combined answer for **(a)** and **(b)**) (Max 2 out of 3) For resolving in both directions M1 For (6cos30°)**i** + (2 x10cos30° – 6cos60°)**j** or any vector equivalent A1		
	(ii)	For using Magnitude = $\sqrt{ans(i)^2 + ans(ii)^2}$	M1	
		Magnitude is 15.2 N ft only following sin/cos mix and for answer 5.66 N	A1 ft	2
3	**(i)**	Region under $v = 2t$ from $t = 0$ to $t = T$ indicated	B1	1
	(ii)	For attempting to set up and solve an equation using area Δ = 16 or for using $s = \frac{1}{2} 2t^2$	M1	
		For $16 = \frac{1}{2} 2T^2$	A1	
		$T = 4$	A1	3
		SR (for candidates who find the height of the Δ but do not score M1) (Max 1 out of 3) For $h/T = 2$ or $h = 2T$ or $v = 8$ B1		

	(iii)	For using distance = 10 × ans **(ii)** or for using the idea that the distance is represented by the area of the relevant parallelogram or by the area of the trapezium (with parallel sides 9 and 4 and height 10) minus the area of the triangle (with base 5 and height 10)	M1	
		Distance is 40m	A1 ft	2
4	**(i)**	For differentiating x	M1	
		$\dot{x} = t + \frac{1}{10}t^2$	A1	
		Speed is 20 ms^{-1}	A1	3
	(ii)	$\ddot{x} = 1 + \frac{1}{5}t$	B1 ft	
		For attempting to solve $\ddot{x}(t) = 2\ddot{x}(0)$ $(1 + \frac{1}{5}t = 2)$	M1	
		$t = 5$	A1	3
5	**(i)**	For resolving forces on any two of A, or B, or A and B combined ($T_1 = W_A + T_2, T_2 = W_B, T_1 = W_A + W_B$)	M1	
		Tension in S_1 is 4 N or Tension in S_2 is 2 N Accept 0.4g or 3.92 (from 9.8 or 9.81) for T_1 Tension in S_2 is 2 N or Tension in S_1 is 4 N Accept 0.2g or 1.96 (from 9.8 or 9.81) for T_2 **SR** (for candidates who omit g) (Max 1 out of 3) T_1 = 0.4 and T_2 = 0.2 B1	B1 A1	3
	(ii)	For applying Newton's second law to A, or to B, or to A and B combined	M1	
		For any one of the equations $T + 2 - 0.4 = 0.2a$, $2 - T - 0.2 = 0.2a$, $4 - 0.4 - 0.2 = 0.4a$	A1	
		For a second of the above equations	A1	
		For solving the simultaneous equations for a and T	M1	
		Acceleration is 8.5 ms^{-2}, tension is 0.1 N Accept 8.3 from 9.8 or 8.31 from 9.81 **SR** (for candidates who obtain only the 'combined' equation) (Max 3 out of 5) For applying Newton's second law to A and B combined M1 For $4 - 0.4 - 0.2 = 0.4a$ A1 Acceleration is 8.5 ms^{-2} A1	A1	5

6	**(i)**	For using $F = \mu R$ and $R = mg$ $(F = 0.025 \times 0.15 \times 10)$	M1	
		Frictional force is 0.0375 N or 3/80 N Accept 0.0368 from 9.8 or 9.81	A1	2
	(ii)	For using $F = ma$ (-0.0375 = 0.15a) or $d = \mu g$	M1	
		Deceleration is 0.25 ms^{-2} (or a = - 0.25) A.G.	A1	2
	(iii)	For using $s = ut + \frac{1}{2}at^2$ $(s = 5.5 \times 4 + \frac{1}{2}(-0.25)16)$	M1	
		Distance *AB* is 20m	A1	2
	(iv)	For using $v^2 = u^2 + 2as$ $(v^2 = 3.5^2 - 2 \times 0.25 \times 20)$	M1	
		Speed is 1.5 ms^{-1} (ft $\sqrt{(24.5 - (iii))/2}$)	A1 ft	2
	(v)	Return dist. = $\frac{3.5^2}{2 \times 0.25}$ or distance beyond *A* = $\frac{(iv)^2}{2 \times 0.25}$	M1	
		Total distance is 44.5 m (ft 24.5 + **(iii)** or 2(**(iv)**^{2} + **(iii)**)	A1 ft	2
7	**(i)**	PE gain = $mg(2.5\sin 60^o)$	B1	
		For using KE = ½ mv^2	M1	
		For using the principle of conservation of energy (½ $m8^2$ - ½ mv^2 = $mg(2.5\sin 60^o)$)	M1	
		Alternative for the above 3 marks: For using Newton's Second Law or stating $a = -g\sin 60^0$ a = -8.66 (may be implied) For using $v^2 = u^2 + 2as$ $(v^2 = 64 - 2 \times 8.66 \times 2.5)$	 M1* A1 M1dep*	
		Speed is 4.55 ms^{-1} Accept 4.64 from 9.8 or 9.81	A1	4
	(ii)	For using ½ mu^2 (>) mgh_{max} (½ 8^2 > $10h_{max}$)	M1	
		For obtaining 3.2m A.G.	A1	2
	(iii)	Energy is conserved or absence of friction or curve *BC* is smooth (or equivalent) and *B* and *C* are at the same height or the PE is the same at *A* and *B* (or equivalent)	B1	1

	(iv)	WD against friction is 1.4 × 5.2	B1	
		For WD = KE loss (or equivalent) used	M1	
		$1.4\times5.2=\frac{1}{2}0.4(8^2-v^2)$ or $1.4\times5.2=\frac{1}{2}0.4((i)^2-v^2)+0.4\times10(2.5\sin60^\circ)$ (12.8 or 4.14 + 8.66)	A1	
		Alternative for the above 3 marks: For using Newton's Second Law $0.4g(2.5\sin60^o\div5.2)-1.4=0.4a$ (a = 0.6636) For using $v^2=u^2+2as$ with u ≠ 0 $(v^2=4.55^2+2\times0.6636\times5.2)$	M1* A1 M1dep*	
		Speed is 5.25 ms^{-1}	A1	4

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GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/05, 8719/05

MATHEMATICS AND HIGHER MATHEMATICS
Paper 5 (Mechanics 2)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

Mechanics 2

1 The distance from the centre to the rod is $\sqrt{25^2 - 24^2}$ B1

For taking moments about the centre of the ring or about the mid-point of the rod, or C.O.M. of frame (correct number of terms required in equation) M1

(1.5 + 0.6)$\bar{x}$ = 0.6 x 7 or (1.5 + 0.6)(7 - $\bar{x}$) =1.5 x 7
1.5$\bar{x}$ = 0.6 (7 - $\bar{x}$) A1

Distance is 2cm A1
SR Allow M1 for 48.7 = (50π + 48) $\bar{x}$

4

2 **(i)** *OQ* = 4 tan 20° (=1.456) B1

OG = 1.5 B1

G not between *O* and *Q* (all calculations correct) B1

3

(ii) Hemisphere does not fall on to its plane face *B1 ft

Because the moment about *P* is clockwise or the centre of mass is to right of *PQ* (dep)* B1 ft

2

3 **(i)** Rope is at 30° to wall, or beam is at 0° to the horizontal or a correct trig. ratio used B1

For taking moments about *A* or
For taking moments about *P* and resolving horizontally M1

2.5*T* = 45*g* x 3cos 30° or
5*H* = 45*g* x 3cos 30° and *H* = *T*sin30° A1 ft

Tension is 468 N A1

4

(ii) Horizontal component is 234 N (ft ½ *T*) B1 ft

For resolving forces vertically (*V* = 45*g* - *T*cos 30°) M1

Magnitude of vertical component is 45 N A1 ft
SR angle incorrect **(i)** B0, M1, A1 ft A0, **(ii)** B1 ft (*T* and angle), M1, A0

3

4 **(i)** For using Newton's second law with $a = v\frac{dv}{dx}$ M1

$-\frac{1}{3v} = 0.2v\frac{dv}{dx}$ A1

$3v^2\frac{dv}{dx} = -5$ from correct working A1

3

(ii) For separating the variables and attempting to integrate M1

$v^3 = (A) - 5x$ A1

For using $x = 0$ and $v = 4$ to find A, and then substituting $x = 7.4$ (or equivalent using limits) M1

$v = 3$ A1

4

5 **(i)** For resolving forces vertically (3 term equation) M1

$T\cos 60^0 + 0.5 \times 10 = 8$ A1

Tension is 6 N A1

3

(ii) Radius of circle is $9\sin 60°$ (7.7942) B1

For using Newton's second law horizontally with $a = \frac{v^2}{r}$ M1

$6 \sin 60^o = 0.5\frac{v^2}{(9\sin 60^\circ)}$ A1 ft

Alternative for the above 2 marks:

For using Newton's second law perpendicular to the string with $a = \frac{v^2}{r}$ M1

$(8 - 0.5 \times 10)\sin 60° = 0.5\frac{v^2}{(9\sin 60^\circ)}\cos 60°$ A1 ft

Speed is 9 ms^{-1} A1

4

NB Use of $mr\omega^2$, the M1 is withheld until $v = r\omega$ is used
SR Lift perpendicular to the string:
(i) $8\sin 60^o = 0.5g + T\cos 60^o \rightarrow T = 3.86$: M1, A1, A1 (-1 MR) (2 out of 3 max);
(ii) $3.86\sin 60^o + 8\cos 60^o = \frac{0.5v^2}{9\sin 60^\circ}$: B1, M1, A1√, A1 (-1 MR) (3 out of 4 max)
$\Rightarrow$ 10.7

6 **(i)** For using $y = \dot{y}_0 t - \frac{1}{2}gt^2$ with $y = 0$ and $t = 10$ or

$\dot{y} = \dot{y}_0 - gt$ with $\dot{y} = 0$ and $t = 5$ M1

$0 = 60\sin\alpha \times 10 - \frac{1}{2} \times 10 \times 10^2$ or $0 = 60\sin\alpha - 10 \times 5$ A1

$\alpha = 56.4°$ A1

3

(ii) For substituting $t = 5$ into $y = \dot{y}_0 t - \frac{1}{2}gt^2$ or $\dot{y} = 0$ into

$\dot{y}^2 = \dot{y}_0^2 - 2gy$ or $\dot{y} = 0$ and $t = 5$ into $y = \frac{\dot{y}_0 + \dot{y}}{2} t$ M1

Greatest height is 125m A1

2

(iii) $\dot{y} = 60\sin\alpha - gT$ B1

$\dot{x} = 60\cos\alpha$ B1

For attempting to solve $\dot{x} = \dot{y}$, or a complete method for an equation in T using $\dot{x} = \dot{y}$ M1

$T = 1.68$ A1

4

NB. Use of $\dot{y}_0 = 60$ in **(i)** and **(ii)** is M0

7 **(i)** For using $T = \frac{\lambda x}{L}$ $\left(\frac{130\times 3}{10} \text{ or } \frac{130\times 1.5}{5}\right)$ M1

Tension is 39 N A1

2

(ii) For resolving forces vertically (mg = 2 x 39 x $\frac{5}{13}$) M1

Mass is 3kg A1

2

(iii) Extension = 20 - 10 (or 10 - 5) B1

For using EPE = $\frac{\lambda x^2}{2L}$
(L must be 10 or 5; must be attempt at extension, e.g. x = 20 or x = 8 - 2.5 is M0)

[EPE = $\frac{130\times 10^2}{2\times 10}$ or EPE = 2 x $\frac{130\times 5^2}{2\times 5}$]
(Allow M1 only for x = 2 or 3) M1

EPE is 650 J (ft attempted extension in lowest position) A1 ft

3

(iv) Change in GPE = 3 x 10 x 8 B1 ft

For using the principle of conservation of energy with KE, GPE and EPE all represented M1

650 = ½3v^2 + 3 x 10 x 8 + $\frac{130\times 2^2}{2\times 10}$ A1 ft

Speed is 16 ms^{-1} A1

4

CAMBRIDGE
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GCE A AND AS LEVEL
AICE

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/06, 0390/06

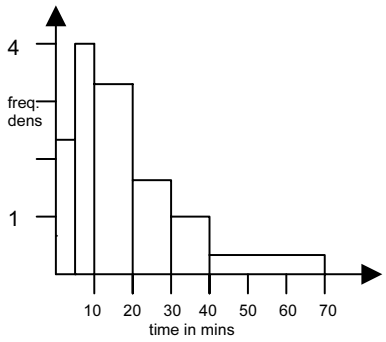
MATHEMATICS
Paper 6 (Probability and Statistics 1)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1	**(i)**	False zero	B1	**1**	Or any sensible answer
	(ii)	**(a)** Stem | Leaf 3 | 45 4 | 145 5 | 02 6 | 2 7 | 339 8 | 344556679 9 | 1 Key 3 | 4 rep 34, or stem width = 10	B1 B1 B1	 **3**	For correct stem, i.e. not 30, 40, 50 etc. For correct leaf, must be sorted For key, NB 30 | 4 rep 34 gets B1 here
		(b) 79	B1 ft	**1**	For correct answer, only ft from a sorted stem and leaf diagram
2	**(i)**	P(N, $\overline{N}$) = $\frac{3}{10}\times\frac{7}{9}$ Mult. By 2 = 7/15 **AG** **OR** Total ways $_{10}C_2$ (= 45) Total 1 of each $_7C_1 \times {}_3C_1$ (= 21) Prob = 21/45 = 7/15 **AG**	M1 A1 M1 A1	 **2** **2**	For multiplying 2 relevant possibilities For obtaining given answer legitimately For both totals For obtaining correct answer
	(ii)	P (N, N) – 3/10 x 2/9 (= 1/15) P($\overline{N}$, $\overline{N}$) = 7/10 x 6/9 (= 7/15) x | 0 1 2 P (X=x) | 7/15 7/15 7/15	M1 M1 B1	 **3**	For 2 correct numbers multiplied together, can be implied For 2 correct numbers multiplied together or subtracting from 1 All correct. Table correct and no working gets 3/3
	(iii)	E(X) = 1 x 7/15 + 2 x 1/15 = 3/5	B1 ft	**1**	For correct answer or equivalent. Only ft if $\sum$ p = 1
3	**(i)**	P(X > 120) = 1 - $\Phi\left(\frac{120-112}{17.2}\right)$ = 1 - Φ (0.4651) = 1 - 0.6790 = 0.321	M1 M1 A1	 **3**	For standardising with or without the $\sqrt{}$, 17.2^2, but no cc. For finding the correct area, 1 – their Φ(z), NOT Φ(1 – their z(0.4651)) For correct answer

	(ii)	z = -0.842 -0.842 = $\frac{103-115}{\sigma}$	B1 M1		For z, ± 0.842 or ± 0.84 For solving an equation involving their z or z = 0.7881 or 0.5793 only, 103, 115 and σ or $\sqrt{\sigma}$ or σ^2, i.e. must have used tables
		σ = 14.3	A1	**3**	For correct answer
4	**(i)**	$(0.7)^{24} \times (0.3)^6 \times {}_{30}C_{24}$	M1		For relevant binomial calculation
		= 0.0829	A1	**2**	For correct answer
		OR normal approx. P(24) = $\Phi((24.5 - 21)/\sqrt{6.3})$ - $\Phi((23.5 - 21)/\sqrt{6.3})$	M1		For subtracting the 2 phi values as written
		= 0.9183 – 0.8404 = 0.0779	A1	**2**	For correct answer
	(ii)	μ = 30 x 0.7 = 21, σ^2 = 30 x 0.7 x 0.3 = 6.3	B1		For 21 and 6.3 seen
		P(< 20) = $\Phi\left(\frac{19.5-21}{\sqrt{6.3}}\right)$ =	M1		For standardising process, must have $\sqrt{}$, can be + or –
		Φ (-0.5976)	M1 M1		For continuity correction 19.5 or 20.5 For using 1 - some area found from tables
		= 1 - 0.7251 = 0.275	A1	**5**	For correct answer
5	**(i)**	${}_6C_3 \times {}_4C_2 = 120$	M1		For multiplying 2 combinations together, not adding, no perms, ${}_{10}C_3 \times {}_{10}C_2$ or ${}_5C_3 \times {}_5C_2$ would get M1
			A1	**2**	For answer 120
	(ii)	${}_6C_4 \times {}_4C_1$ (= 60)	M1		For reasonable attempt on option 4M 1W, or 5M, 0W, can have + here and perms
		${}_6C_5 \times {}_4C_0$ (= 6)	M1		For other option attempt
		Answer = 186	A1	**3**	For correct answer
	(iii)	Man and woman both on ${}_5C_2 \times {}_3C_1$ (= 30)	M1		For finding number of ways of the man and woman being on together, need not be evaluated but must be multiplied
		120 - 30 = 90	M1		For subtracting a relevant number from their **(i)**
			A1	**3**	For correct answer

		OR $_5C_2 \times {}_3C_2$ (= 30) $_3C_1 \times {}_5C_3$ (= 30) $_5C_3 \times {}_3C_2$ (= 30) $\sum$ = 90	M1 M1 A1	**3**	Any 2 of man in, woman out Woman in, man out Neither in
		OR $_3C_1 \times {}_5C_3$ (= 30) $_3C_2 \times {}_6C_3$ (= 60) $\sum$ = 90	M1 M1 A1	**3**	Woman in, man out Woman out, any man For correct answer
		OR $_5C_2 \times {}_3C_2$ (= 30) $_5C_3 \times {}_4C_2$ (= 60) $\sum$ = 90	M1 M1 A1	**3**	Man in, woman out Man out, any woman For correct answer
6	**(i)**	P(*G*) = number of g'parents/total people = 6/16 = 3/8	M1 A1	**2**	For appreciating total g'parents/total people, can be implied For correct answer
	(ii)	P(H1, *G*) + P(H2, *G*) + P(H3, *G*) $=\frac{1}{3}\times\frac{2}{7}+\frac{1}{3}\times\frac{3}{7}+\frac{1}{3}\times\frac{1}{2}=\frac{17}{42}$ (= 0.405)	B1 M1 A1	**3**	For any correct 2-factor product, need not be evaluated For addition of 3 relevant 2-factor products For correct answer or equivalent
	(iii)	P(H1 | *G*) + P(H2 | *G*) $=\frac{2/21}{17/42}+\frac{3/21}{17/42}=\frac{10}{17}$ **OR** P(H3 | *G*) = 7/17 Answer = 1 - 7/17 = 10/17	M1 M1 A1 A1 M1 M1 A2	**4**	For summing exactly 2 probability options For dividing by answer to **(ii)**, only if not multiplied as well, and p must be < 1 For one correct probability For correct answer or equivalent For finding prob. options no parents For subt. from 1 For correct answer
7	**(i)**	Mean = (2.5 x 11 + 7.5 x 20 + 15 x 32 + 25 x 18 + 35 x 10 + 55 x 6)/97 = 18.4	M1 M1 A1		For using their mid-intervals (not end points or class widths) For using $\frac{\sum fx^2}{\sum f}$ any *x* For correct answer, cwo, 18.4 no wkg 3/3

		sd = $\sqrt{}$ (2.5^2 x 11 + 7.5^2 x 20 + 15^2 x 32 + 25^2 x 18 + 35^2 x 10 + 55^2 x 6)/97 - mean^2) = 13.3	M1 A1	**5**	For using $\frac{\sum fx^2}{\sum f}$ - (their mean)^2 or equivalent, no $\sqrt{}$ needed, not $(\sum fx)^2/\sum f$ For correct answer
	(ii)	Freq. densities: 2.2, 4.0, 3.2, 1.8, 1.0, 0.2  	M1 A1 B1 B1	**4**	For attempting a frequency density of some sort (or scaled frequency), can be upside down but not multiplied For correct heights on the graph For correct bars on uniform horiz. scale, i.e. from 0 to 5 etc. Freq. density or scaled freq. labelled on vertical axis, time or mins on horiz., 'class width' is not enough

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

June 2003

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/07, 8719/07

MATHEMATICS AND HIGHER MATHEMATICS
Paper 7 (Probability and Statistics 2)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1 (i) 2.5 1.25	B1 B1	**2**	For correct mean. For correct variance
(ii) 5 5	B1ft B1ft	**2**	For correct mean. For correct variance
2 $H_0 : p = 0.6$ $H_1 : p > 0.6$	B1		For correct H_0 and H_1
$P(X \geq 10) = {}_{12}C_{10}0.6^{10}0.4^2 + {}_{12}C_{11}0.6^{11}0.4^1 + 0.6^{12}$	M1*		For one Bin term (n = 12, p = 0.6)
$= 0.0834$	M1*dep		For attempt $X = 10, 11, 12$ or equiv.
	A1		For correct answer (or correct individual terms and dig showing 0.1)
Reject H_0, i.e. accept claim at 10% level	B1ft	**5**	For correct conclusion
S.R. Use of Normal scores 4/5 max	B1		For correct H_0 and H_1
$z = \frac{9.5 - 7.2}{\sqrt{2.88}}$ (or equiv. Using N(0.6, 0.24/12)) $= 1.3552$	M1		Use of N(7.2, 2.88) or N(0.6, 0.24/12) and standardising with or without cc
$Pr(>9.5) = 1 - 0.9123 = 0.0877$	A1		For correct answer or 1.3552 and 1.282 seen
Reject H_0, i.e. accept claim at 10% level	B1ft		For correct conclusion
3 (i) $31 \pm 2.326 \times \frac{3}{\sqrt{20}}$	B1		For correct mean
$= (29.4, 32.6)$	M1		Calculation of correct form $\bar{x} \pm z \times \frac{s}{\sqrt{n}}$ (must have $\sqrt{n}$ in denominator)
	B1		$z = 2.326$
	A1	**4**	Correct answer
(ii) 30% is inside interval	ftB1*		
Accept claim (at 2% level)	ftB1*dep	**2**	S.R. Solutions not using (i) score B1ft only for correct working and conclusion
4 (i) $P(X > 1.5) = \left[x - \frac{x^2}{4}\right]_{1.5}^{2}$	M1		For substituting 2 and 1.5 in their $\int f(x)dx$ (or area method ½ their base x their height)
or $1 - \left[x - \frac{x^2}{4}\right]_{.0}^{1.5}$			
$= 0.0625$	A1	**2**	For correct answer

(ii)	E(X) = $\int_0^2 (x - \frac{1}{2}x^2)dx = \left[\frac{x^2}{2} - \frac{x^3}{6}\right]_0^2$	M1		For evaluating their $\int xf(x)dx$
	= 2/3	A1	**2**	For correct answer
(iii)	$m - \frac{m^2}{4} = 0.5$	M1		For equating their $\int f(x)dx$ to 0.5
		M1		For solving the related quadratic
	$m = 0.586$ (2-$\sqrt{2}$)	A1	**3**	For correct answer
5 **(i)**	P(X < 1.7) = $\Phi\left(\frac{1.7-2.1}{0.9/\sqrt{20}}\right)$	B1		For identifying prob Type I error
		M1		For standardising
	= 1 - Φ (1.9876)	A1		For correct standardising and correct area
	= 0.0234	A1	**4**	For correct final answer
(ii)	P(Type II error) = P(X > 1.7)	B1		For identifying prob for Type II error
	= 1 - $\Phi\left(\frac{1.7-1.5}{0.9/\sqrt{20}}\right)$	M1		For standardising using 1.5 and their 1.7
		A1		For correct standardising and correct area
	= 1 - Φ (0.9938) = 0.160	A1	**4**	For correct final answer
6 **(i)**	λ = 1.25	M1		For attempting to find new λ and using it
	P(X < 4) = $e^{-1.25}\left(1 + 1.25 + \frac{1.25^2}{2} + \frac{1.25^3}{6}\right)$	M1		For summing P((0,) 1, 2, 3) or P(0, 1, 2, 3, 4) using a Poisson expression
	= 0.962	A1	**3**	For correct answer
(ii)	X ~N(182.5, 182.5)	B1		For correct mean and variance
	P(> 200 breakdowns) = 1 - $\Phi\left(\frac{200.5-182.5}{\sqrt{182.5}}\right)$	M1		For standardising process with or without continuity correction
	= 1 - Φ (1.332)	A1ft		For correct standardising and correct tail
	= 0.0915 (0.0914)	A1	**4**	For correct answer
(iii)	λ = 5 for phone calls λ = 6.25 for total	B1		
	P(X = 4) = $e^{-6.25}\left(\frac{6.25^4}{4!}\right)$	M1		For summing their two λ s and using a Poisson expression OR alt. method using sep. distributions 5 terms req.
	= 0.123	A1	**3**	For correct answer

7	**(i)** 20 of A ~A* ~N(401, 20 x 0.15^2) ~N(401, 0.45)	B1		For correct mean for either
	20 of B ~B* ~N(401, 1.458)	B1		For variance 20 x 0.15^2 or 20 x 0.27^2
	$A^* - B^*$ ~N(0, 1.908)	M1		For adding their two variances
	P($A^* - B^* > 2$)			
	$= 1 - \Phi\left(\frac{2-0}{\sqrt{1.908}}\right)$	M1		For consideration of their $A^* - B^* > 2$
	$= 1 - \Phi(1.4479)$	M1		For standardising and finding correct area
	$= 0.0738$	A1	**6**	For correct answer
	OR $\overline{A}$ ~N(20.05, 0.15^2/20), $\overline{B}$ ~N(20.05, 0.27^2/20)	B1		For correct mean for either
		B1		For variance $0.15^2/20$ or $0.27^2/20$
	$\overline{A} - \overline{B}$ ~N(0, 0.00477)	M1		For adding their variances
	P($\overline{A} - \overline{B} > 0.1$)	M1		For consideration of their $\overline{A} - \overline{B} > 0.1$
	$= 1 - \Phi\left(\frac{0.1-0}{\sqrt{0.00477}}\right)$	M1		For standardising and finding correct area
	$= 0.0738$	A1	**6**	For correct answer
	(ii) $1.96 = \frac{20.07 - 20.05}{(0.15/\sqrt{n})}$	M1		For an equation of correct form on RHS involving $\sqrt{n}$
		B1		For 1.96 used
		M1		For solving an equation of correct form (any z)
	$n = 216$	A1	**4**	For correct answer